1. $\Delta u = 2\cos 2\varphi$, $r > 2$,
   $u(2,\varphi) = \cos\varphi$.
2. $\Delta u = 0$, $0 < x < \pi$, $0 < y < \pi$, если $u_x(0, y) = 0, u_x(\pi, y) = 0, u_y(x,0) = \cos 3x, u_y(x,\pi) = 0$.
3. $\Delta u = 0, -\infty < x < \infty, -2\pi < y < 0; u(x,0) = 0,$
   $u(x,-2\pi) = 1,$ при $x < 1; u(x,-2\pi) = 0,$ при $x \geq 1$
4. $$\begin{cases} u_{tt} = u_{xx} + \cos x, t > 0, 0 < x < \pi; \\ u_x(0,t) = 0, \quad u_x(\pi,t) = \pi, \\ u(x,0) = 4\cos 5x + \dfrac{x^2}{2}, \\ u_t(x,0) = 1. \end{cases}$$
5. $u_{tt} = 4u_{xx}, 0 < x < \infty, t > 0$

$u_x(0,t) = 0$, $u(x,0) = \begin{cases} \sin x, x \in [0;\pi] \\ 0, x \notin [0;\pi] \end{cases}$, $u_t(x,0) = 0$. Найти $u(3\pi/4, t)$ и нарисовать график.

6. $u_{tt} = 9u_{xx}, t > 0, x > 0;$
   $u(0,t) = 0 \quad u(x,0) = \sin x \quad u_t(x,0) = 3\cos x.$

Описать процесс колебаний. Построить профиль струны в момент времени $t = \dfrac{3\pi}{4}$.

$$\begin{cases} \Delta u = 2\cos 2\varphi, & r > 2 \\ u(2,\varphi) = \cos\varphi \end{cases}$$

$u = 2R(r)\cos 2\varphi$

$\Delta u = 2\left(r^2R'' + rR' - 4R\right)\cos 2\varphi = 2\cos 2\varphi$

$\Leftrightarrow r^2R'' + rR' - 4R = r^2$

Решаем однородное:

$\lambda(\lambda-1)+\lambda-4=0 \Rightarrow \lambda = \pm 2 \Rightarrow R_{общ} = C_1e^{2t} + C_2e^{-2t}$

частное в виде ate^{2t}

Характеристическое у-ие: $y'' - 4y = e^{2t}$

$\Rightarrow a(e^{2t} + 2te^{2t})' - 4ate^{2t} = e^{2t}$

$\Leftrightarrow 2ae^{2t} + 2ae^{2t} + 4ate^{2t} - 4ate^{2t} = e^{2t}$

$\Rightarrow A = \frac{1}{4} \Rightarrow \frac{1}{4}te^{2t}$

$u_r = \frac{\ln r \cdot r^2}{4}\sin 2\varphi$

$u = u_1 + \frac{\ln r \cdot r^2}{4}\sin 2\varphi$

$\Delta u_1 = 0,\ r > 2$

$u(2,\varphi) = u_1(2,\varphi) + \ln 2\sin 2\varphi$

$u_1(2,\varphi) = \cos\varphi - \ln 2\sin 2\varphi$

Для внешней части круга.

$u = A + \sum r^{-n}(A_n\cos n\varphi + B_n\sin n\varphi)$

$u(2,\varphi) = \cos\varphi - \ln 2\sin 2\varphi$

$A = 0,\ A_1 = 2,\ B_1 = 0$

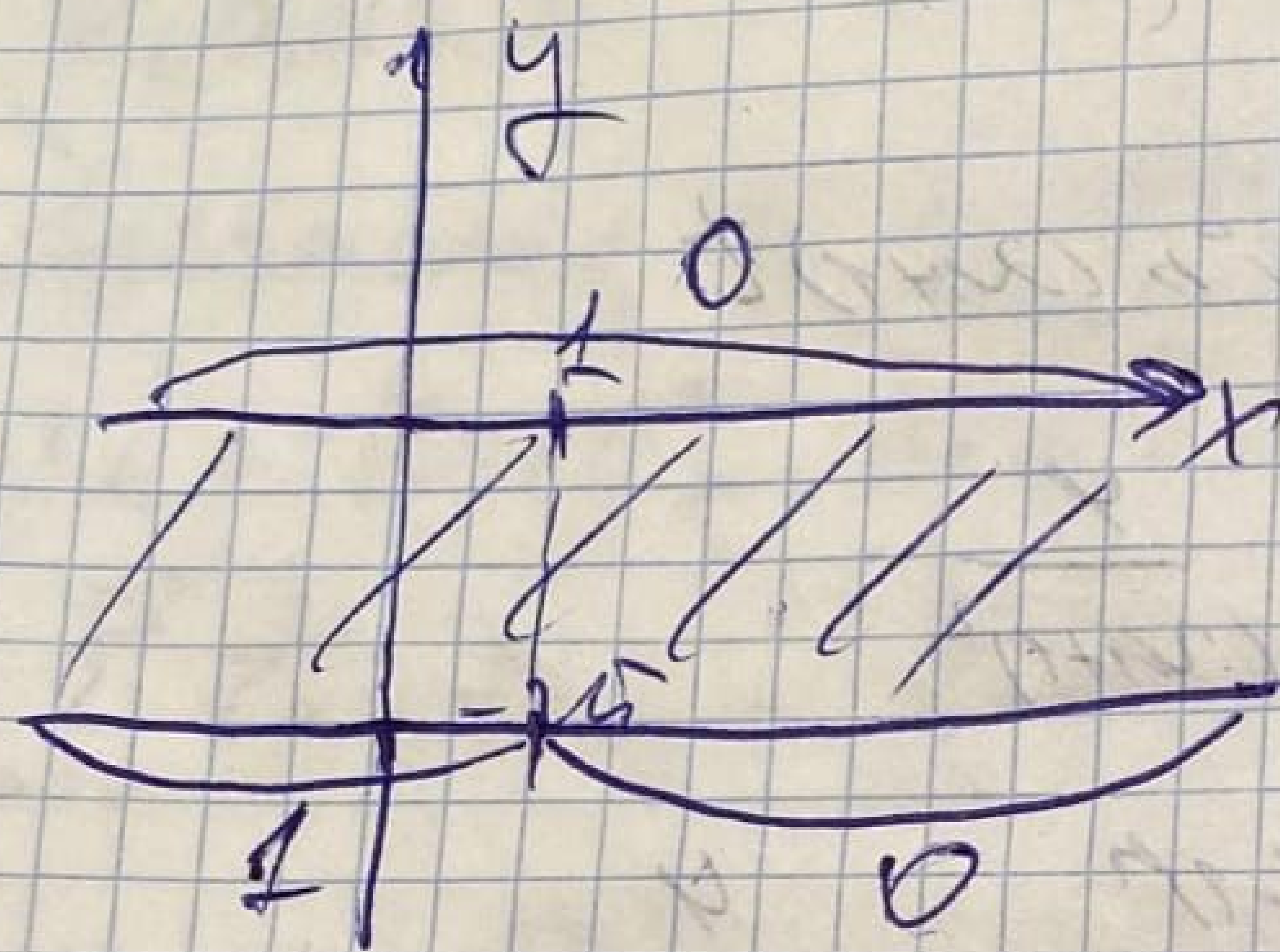
$B_2 = -\ln 2 \cdot 4$

$u(x,t) = \underline{r^{-1}\cdot(2\cos\varphi) - r^{-2}\cdot 4\cdot\sin 2\varphi\cdot\ln 2}$

Ответ: $\ldots + \frac{\ln r \cdot r^2}{4}\sin 2\varphi$

Вар. 2, №3

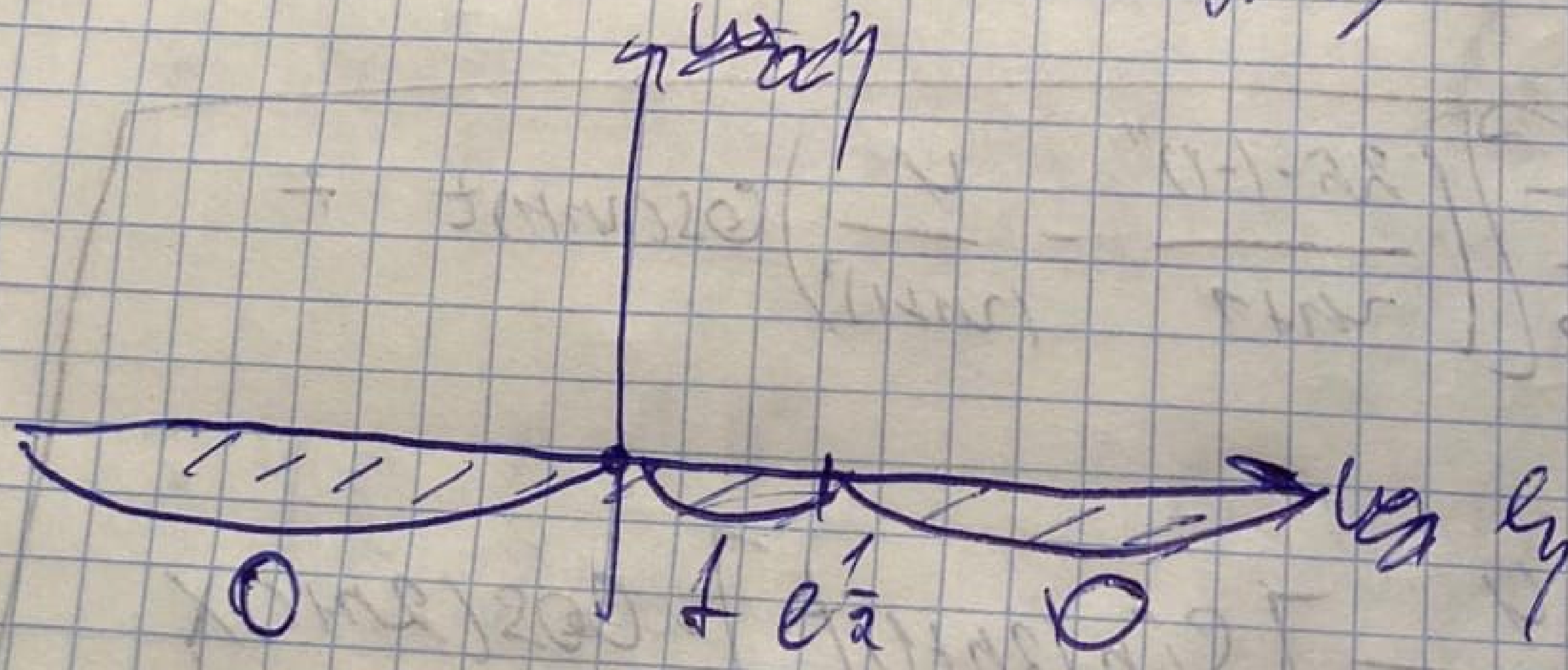
$$\begin{cases}\Delta u = 0,\ -\infty < x < \infty,\ -2\pi < y < 0,\ u(x,0)=0\\ u(x,-2\pi) = 1,\ x < 1\\ u(x,-2\pi) = 0,\ x > 1\end{cases}$$



$\to w_1(z) = z + 2\pi i$

$$w_2(w_1) = e^{\frac{1}{2}z + \pi i} = e^{\frac{x}{2}}(\cos y_2 + i\sin(y/2))\cdot(-1)$$

$$= -e^{\frac{x}{2}}(\cos(y/2) + i\sin(y/2))$$



~~$w_2(0,y) =$~~

$$w_2(x,0) = -e^{\frac{x}{2}} \qquad \xi = -e^{\frac{x}{2}}\cos(y/2)$$

$$\eta = -e^{\frac{x}{2}}\sin(y/2)$$

$$w_2(x,-2\pi) = e^{\frac{x}{2}}$$

$$\Rightarrow F(\xi_1) = \begin{cases} 1, & 0 \le \xi_1 < e^{\frac{1}{2}} \\ 0, & \text{иначе.} \end{cases}$$

$$u(z_0) = \frac{\eta_0}{\pi} \int_{-\infty}^{\infty} \frac{F(\xi_1)}{(\xi_1-\xi_0)^2+\eta_0^2} = \frac{\eta_0}{\pi} \int_{0}^{e^{\frac{1}{2}}} \frac{d\xi_1}{(\xi_1-\xi_0)^2+\eta_0^2} =$$

$$= \frac{1}{\pi}\left[\operatorname{arctg}\left(\frac{e^{\frac{1}{2}}-\xi_0}{\eta_0}\right) - \operatorname{arctg}\left(-\frac{\xi_0}{\eta_0}\right)\right] =$$

$$= \frac{1}{\pi}\left[\operatorname{arctg}\left(\frac{e^{\frac{1}{2}}+e^{\frac{x_0}{2}}\cos(\frac{y_0}{2})}{-e^{x_0/2}\cdot\sin(y_0/2)}\right) - \operatorname{arctg}\left(\frac{e^{\frac{x_0}{2}}\cos(y_0/2)}{e^{\frac{x_0}{2}}\sin y_0/2}\right)\right]$$

$$\Rightarrow \boxed{u(x,y) = \frac{1}{\pi}\left[\operatorname{arctg}\left(\frac{e^{\frac{1}{2}}+e^{\frac{x}{2}}\cos(y/2)}{-e^{x/2}\sin(y/2)}\right) - \operatorname{arctg}\left(\frac{e^{x/2}\cos(y/2)}{-e^{x/2}\sin y/2}\right)\right]}$$